MADHAVA MATHEMATICS COMPETITION- A RECENT INITIATIVE IN INDIA

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Abstract. This article presents a survey of the Madhava Mathematics Competition, a recently started Mathematics competition for undergraduate students in India. The competition was started in the academic year 2009-10 and has received a tremendous response in last eight years. it can be seen as an extension of the Olympiad competitions to undergraduate classes. We therefore take a review of Olympiads in India and then discuss the various aspects of this new competition.

1. Introduction

In this article, we propose to present a recent initiative in India in the field of Mathematical Competitions. We shall present an account of Madhava Mathematics Competition- a mathematics competition for the students of undergraduate classes. The crux of the presentation lies in the discussion of the problems used for the competition and responses of the students in terms of some beautiful solutions to these problems. In the process, we shall also discuss the overall Indian scenario in the context of mathematics education at the school and college levels, the decisive role of this competition in enhancing the mathematical abilities of students, and some statistics, depicting the trends in the competition. A feedback of the competition from the student fraternity, teachers and senior mathematicians is an integral part of the article.

In section 2, a survey of the impact of Mathematical Olympiad in India has been taken. The Mathematical Olympiad has provided a necessary impetus and motivation for launching the Madhva Mathematics Competition. In section 3, we shall focus on the evolution of Madhava Competition, its growth and the nity-gritties of organising the competition at the national level. This section also includes a brief Bio-Mathography of the 14th century Indian mathematician Madhava. In section IV, some sample problems that appeared in the competition along with their solutions will be discussed. Some elegant solutions devised by the students while writing the competition have also been included. We shall see that a comparative analysis of the understanding of students in variety of topics at the undergraduate level is extremely

Key words and phrases. Madhava Mathematics Competition, Mathematical Olympiads,

illuminating. Section V is devoted to description of Nurture Camps-a distinctive feature of the competition. In Section VI, a feedback of participants and mathematics teachers involved in the competition as well as comments of senior mathematician of the country have been included. In section VII, we conclude with an epilogue and the future plans of expansion.

2. Mathematical Olympiads- A Precursor to Madhava Competition

India started participating in the International Mathematical Olympiad (IMO) in 1989. Following a few years of its being based in Bangalore, the nodal center of the activity was shifted to the Homi Bhabha Center for Science Education(HBCSE), Mumbai which is now in-charge of the competitions at all levels. The selection of the Indian team for IMO takes place in three stages. A regional level competition, called Regional Mathematical Olympiad (RMO) is conducted in about twenty five regions and then thirty students from each region are selected to participate in the Indian National Mathematical Olympiad (INMO). Only those students who are selected in RMO and those who have received an INMO certificate of merit are eligible to appear for the INMO provided they are in class XI or below. On the basis of the INMO, the top 30 students in merit from all over the country are chosen as INMO awardees. In addition to INMO awardees, the next 45 students who are in grade XI or lower and have done well in INMO, but have not qualified as INMO awardee are awarded INMO certificate of merit. These students are eligible to appear for INMO of the next year directly without qualifying through RMO , provided they are not in grade XII.

The INMO awardees are invited for a month long training camp in April-May each year at HBCSE, Mumbai. The INMO awardess of the previous years who are eligible for IMO 2016 and, in addition, who have satisfactorily gone through postal tuition throughout the year, are invited to the training camp as senior students.The junior students receive INMO certificate and a prize in the form of books.The senior students receive a prize in the form of books and cash. On the basis of a number of selection tests during the Camp, a team of the best six students is selected from the combined pool of junior and senior batch participants.

We now present sample problems, one from each of the tests- RMO, INMO and Selection Test.

Problem 2.1 : RMO Problem

[1] Find all integers k such that all the roots of the following polynomial are also integers:

$$f(x) = x^3 - (k-3)x^2 - 11x + (4k-8)$$

Solution 1. Suppose that for some value of k, all the roots of $f(x)$ are integers. We observe that the coefficient of k in the expression of the polynomial is $(-x^2+4)$; meaning that for $x=2$ and $x=-2$, the value of the polynomial does not depend on k.

We get: $f(-2) = 18$ which is positive; and $f(2) = -10$ which is negative. So at least one root lies between -2 and 2.

Case 1: One of the roots is -1. This implies $f(-1) = 3k+5 = 0$; so $k = -\frac{5}{3}$, which is not an integer.

Case 2: One of the roots is 0. This implies $f(0) = 4k-8 = 0$; implying $k=2$. In this case, the polynomial is: $f(x) = x^3 + x^2 - 11x = x(x^2+x-11)$. But the quadratic expression inside the bracket does not have integer roots.

Case 3: One of the roots is 1. This implies $f(1) = 3k-15 = 0$; implying $k=5$. In this case, the polynomial is $f(x) = x^3 - 2x^2 - 11x + 12 = (x-1)(x^2-x-12) = (x-1)(x-4)(x+3)$. So the roots of the polynomial are $1, 4, -3$ which are all integers, as required.

Hence, the only solution is $k=5$; giving $f(x) = x^3 - 2x^2 - 11x + 12$ with roots $1, 4$ and -3.

Solution 2. Consider the polynomial $g(x) = f(x+2)$

If the roots of $f(x)$ are p, q, r, then the roots of $g(x)$ are $p-2, q-2, r-2$.

Also, we note that the constant term of $g(x)$ is equal to $g(0) = f(2) = -18$; and its leading coefficient is still 1.

Hence the product of the roots of $g(x)$ is $(p-2)(q-2)(r-2) = -18$ ..(1)

Since p, q, r are all integers, so are $(p-2), (q-2), (r-2)$. For each possible factorization of (1), we will only check if p, q, r satisfy the correct relationship with the coefficient of x in $f(x)$, or in other words, whether $pq + qr + rp = -11$

[1]The problem is designed by Prashant Sohani, Regional Coordinator, Math Olympiad and Bronze Medalist in IMO in the year 2008

Accordingly, we get the following cases:

$(p+2, q+2, r+2)$	$pq + qr + rp$
$(1, 1, -18)$	41
$(1, -1, 18)$	-61
$(1, 2, -9)$	11
$(1, -2, 9)$	-31
$(-1, 2, -9)$	33
$(1, 3, -6)$	-1
$(1, -3, 6)$	-19
$(-1, 3, 6)$	-11
$(2, -3, 3)$	-5
$(-2, 3, 3)$	-7

We see that only the case of $(-1, 3, 6)$ satisfies the requirement of $pq+qr+rp = -11$. It corresponds to the values of p, q, r as $-3, 1, 4$, and $f(x) = x^3 - 2x^2 - 11x + 12$. Importantly, there exists a value of k, namely $k = 5$, that yields this polynomial. Thus $k = 5$ is the only solution.

The reader must have observed that the first solution is more elegant for it makes a crucial observation about the coefficient of k. Once that is done, the problem becomes considerably easy ! On the contrary, the second solution is somewhat routine.

Problem 2.2 : INMO Problem

Let ABC be a right angled triangle with $\angle B = 90^0$. Let AD be the bisector of $\angle A$ with D on BC. Let the circumcircle of triangle ACD intersect AB again in E and let the circumcircle of triangle ABD intersect AC again in F. Let K be the reflection E in the line BC. Prove that $FK = BC$.

The reader is urged to solve the problem. (Hint: Show that $ACDE$ and $AFDB$ are cyclic quadrilaterals. Alternatively, one may also observe that K, D, F are collinear and further triangles AKF and ABC are similar.

Problem 2.3 : Shortlisted for IMO [2]

(1) There are n circles drawn on a piece of paper in such a way that any two circles intersect in two points and no three circles pass through the same point. Turbo the snail slides along the circles in the following fashion: Initially he move on one of the circles in clockwise direction. Turbo always keep sliding along the current circle until he reaches intersection with another circle. Then he continues his journey along this new circle and also changes the direction of moving i.e. from clockwise to anticlockwise or *vice versa*.

Suppose that Turbo's path entirely covers all circles. Prove that n must be odd.

[2]Both the problems are constructed by Dr. Tejaswi, ex-member of Math Olympiad Cell in India

(2) Let r be a positive integer and $a_0, a_1, \dots$ be an infinite sequence of real numbers. Assumenthat for all no-negative integers m and s , there exists a positive integer n in $[m+1, m+r]$ such that

$$a_m + am + 1 + \cdots a_{m+s} = a_n + an + 1 + \cdots a_{n+s}.$$

Prove that the sequence is periodic i.e. there exists some $p \geq 1$ such that $a_{n+p} = a_n$ for all $n \geq 0$.

The solutions of the shortlisted problems are not included here and interested reader may refer to the official website of IMO.

In the last 28 years, the Olympiad competitions have become very prestigious and competitive in India. The impact of Olympiads in the country has been multifold. The competitions provide a rigorous exposure to students with a very high mathematical aptitude and help in generating more interest in the subject. The difficulty level of problems in the Olympaid contests, as compared to that of problems appearing in their school examinations, is considerably high. As a result, bright students are motivated by this challenge and end up learning the mathematics with more zeal and enthusiasm. On the other hand, a large number of students who appear for RMO but do not necessarily make it to further levels, still learn more mathematics than what their school curriculum demands. As a consequence, the general mathematical know-how at the school level in the country has consistently increased in the post-Olympiad period. The Olympiad movement has evolved as one of the most treasured educational instrument in mathematical circles at school level. Yet another decisive contribution of Olympiad contests has been the strengthening of mathematical abilities of teachers who were involved in training the Olympiad students. In India, the knowledge of the mathematics teachers in school is generally restricted to the school curriculum which essentially deals with basic algebra, geometry and routine arithmetic involving applications in everyday life. Thus, these teachers are generally not in position to tackle problems in Number Theory and Combinatorics and also more advanced Olympiad topics in algebra and geometry. The pool of teachers working for the cause of Olympiad therefore, mainly consists of teachers teaching in undergraduate colleges.

3. Madhava Mathematics Competition: Concept and Scope

Before we proceed to discuss about Madhava Mathematics Competition in detail, the reader, we hope, would be delighted to know about the 14th Century Indian Mathematician Madhava and his work in nutshell.

3.1. **Madhava -The Inventor of Calculus.** **Madhava** is regarded as the founder of the most influential mathematical tradition in India that began in the middle of fourteenth century and continued for about next 250 years. Madhava and his disciples were from Kerala (on the south-west coast of India) and their writings are mainly in Sanskrit and local vernacular language, Malayalam. The Madhava school is a typical example of Indian 'Guru-Shishya Parampara' characterized by flow of knowledge through a chain of teachers and their disciples, from one generation to the next. The only known pupil of Madhava was Parameswara, a very prolific mathematician and an authority in Astronomy. He wrote about twenty-five texts on astronomy and was known for his contributions in eclipse observations. The tradition continued with Parameswara's son Damodara and his disciples Nilakantha and Jyesthadeva. Nilakantha is known for his work 'Tantra-sangraha' and Jyesthadeva for his masterly text 'Yuktibhasa'. Their pupil Sankara wrote a commentary based on Yuktibhasa. All these works mention the contributions of Madhava. **The major achievements of Madhava are astonishing. In particular, one can trace back the invention of Calculus and its applications to trigonometric functions in Madhava's work**.
Some of his major accomplishments have been listed below:

(1) **Madhava-Leibniz Series for π**

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

The Madhava-Leibnitz series being slow in converging, and hence not useful in computing π, an ingenious sequence of correction terms for its partial sums was introduced, using which the computation could be effected efficiently.

(2) **Arc-tangent Series** For $0 \leq \theta \leq \frac{\pi}{4}$,

$$\theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} - \cdots$$

(3) **Madhava's Numerical Value for π** A numerical value of π as found by Madhava is 3.14159265359 which is correct to eleven decimal places. In fact they has a great fascination for determining values of π to great accuracy.

(4) **Series expansion for Sine and Cosine**

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots$$

It is extremely illuminating and interesting to understand the recursive methods employed by the mathematicians of the Kerala School to arrive at these and other such results, especially in the light of the fact that the work was carried out in the Pre-Newtonian period. For a comprehensive account on the mathematics of Kerala School and in particular the contributions of Madhava, the reader is refereed to [?],[?],[?]

We now present a detailed account of the establishment of Madhava Mathematics Competition and its emergence as a national level competition at the undergraduate level.

3.2. Mathematics Competitions: A Prologue to Madhava Competition.

In the cultural history of any society, it is observed that several artists, authors, musicians, athletes, etc. have started their careers with a competition at an early age. In the realm of Science, mathematics included, competitions held at early age help in generating curiosity and interest in the subject and triggers the mind for pursuing intellectual quest. The competitions allow students to stretch their capacities and go beyond the regular curriculum. We sincerely feel that rather than the end result of a competition, the preparation for the competition has far greater value in developing a permanent interest in the subject. Competitions, taken in right spirit, teaches one to face competitive situations in life and inculcates the healthy spirit of accepting defeats with an open mind as well as enjoying the success in a dignified manner. An ultimate objective of a mathematics competition is to provide a platform for students, outside the regular structure of teaching-learning processes that would allow them to become good at mathematics. Thus a well designed Math Competition is certainly an effective educational tool and assumes a very high potential in creating a society with a better perspective for mathematics. Several mathematicians and educationists have therefore taken a keen interest in organizing a good math competition at all stages of learning mathematics. There are literally thousands of math competitions that take place across the globe. The nature of these competitions display a huge variety in terms of level of mathematics, objective type or writing solutions of problems, online or paper based, regional, national or international, prize money involved, supported by government or not, etc. In this labyrinth of mathematics competitions, we shall now localize to Indian scenario and set up a context for introducing Madhava Mathematics Competition for undergraduate students in India.

India stands for pluralism in terms languages, food, dress codes, cultural ethos, etc. and educational system is not an exception. The country is divided into 29 states with each state having their own School Boards that governs school education in the state. There are a few central boards that cater to schools across the country and implement a uniform curriculum in all states. Students have a choice to take their school education either in English or in a regional language as also to chose a state board or a central board. This multi-parametric educational system poses several challenges in designing a competition at a national level. Many states in the country have association of mathematics teachers or mathematical forums that conduct local level mathematics competitions. For example, in the state of Maharashtra in India, Mathex competitions are being conducted across the state for last 50 years. These competitions have gained a very good reputation and many students have been motivation to study more mathematics through these competitions. These local level competitions are very crucial in preparing the mind-set of students (and parents) for reaching out to competitive examinations beyond school horizons. Many of these competitions are held at early stages of schooling. In last 25 years, these local competitions have began to emerge as a precursor to Math Olympiad competitions. As mentioned earlier, Olympiad competitions, at both regional and national levels, have attracted the attention of school students and their parents in a substantial way. A success in Olympiads is considered to be a remarkable achievement in the career of a student and are justly proud of their accomplishment ! The name, fame and charisma of Olympiads is phenomenal and the obvious reason for this growing impact is the high quality of intriguing mathematical problems offering students an opportunity to scratch their brain and bring in their innovation to arrive at beautiful solutions. This excitement of disentangling the knot (or knots !) in a problem is the crux of the Olympiad mathematics. One of the main motives for extending the Olympiad competition from school to undergrad level is to retain the continuity of experiencing the charm of solving challenging problems and therefore generate enthusiasm and love for the subject the among students at a right level, from where they would probably take up mathematics as their life-time intellectual pursuit.

3.3. **The Madhava Mathematics Competition: Operational Aspects.** The Madhava Mathematics Competition was started in the academic year 2009-10, 20 years after India first participated in IMO. In India, XIIth grade is a crucial year for students in the sense that after XIIth grade they have variety of options to go for professional courses such as Engineering, Architecture, Medicine, Law, Management etc. These courses are job oriented and generally students aspire to get admission to one of these professional courses. Most of the students who shine in Olympiad competitions prefer to join elite institutes such as Indian Institute of Technology (IIT). As a result, students entering into undergraduate stream and not getting into professional courses are generally thought of as academically poor. Of course, in recent years the trend is encouraging because many Olympiad toppers are opting to for career in pure sciences through the opportunities provided by the national level elite institutions such as Indian Institute of Science (IISc), Chennai Mathematical Institute (CMI), etc. All these undergrads pursuing pure sciences, especially those who are interested in mathematics were in a way deprived of having math competition like Olympiads. The Madhava Competition has attempted to remedy the situation in a modest way.

The commitment, enthusiasm and drive of teachers of Department of Mathematics, S. P. College, Pune, led to the enterprise of initiating a mathematics competition at the undergraduate level. At this point the reader should note that S. P. College, Pune is a hundred year old, highly reputed educational institute in India catering to the needs and aspirations of more than 7000 students. These teachers submitted a proposal for financial assistance to the National Board for Higher Mathematics (NBHM), a board of mathematicians monitoring the funding of several activities across the country. The NBHM readily agreed to support the activity. Professor S. G. Dani, a renowned mathematician from TIFR and then Chairman of NBHM took keen interest in promoting such a competition. In fact, he named the competition as Madhava Mathematics Competition with a view to spread the name and work of Madhava.

As a pilot project in the first year, the competition was launched in two cities Pune and Mumbai, both located in the state of Maharashtra in the western part of the country. Right in the first year, the competition received very good response , This prompted the organizers to expand it outside the state of Maharashtra and turn it into a National level competition. Any expansion plan certainly demands a lot of active manpower and fortunately, many college teachers came forward and coordinated the event in their regions. The following table indicates the growth of the competition in diverse parts of the country:

Madhava Competition - No of appeared students

Year	2009 - 2010	2010 - 2011	2011 - 2012	2012 - 2013	2013-2014	2014-2015	13 th Dec. 2015
Date of Exam.	3/1/2010	9/1/2011	8/1/2012	6/1/2013	5/1/2014	4/1/2015	13/12/2015
Total No. of students appeared for exam.	820	2644	3880	5151	7672	8327	9041
No. of Regions	2	6	12	17	18	17	19
Regions	Pune	Pune	Pune	Nanded	Nanded	Ahmedabad	Ahmedabad
	Mumbai	Mumbai	Mumbai	Hyderabad	Hyderabad	Ahmednagar	Ahmednagar
		Ahmednagar	Ahmednagar	Kolkata	Kolkata	Allahabad	Allahabad
		Nasik	Nasik	Allahabad	Allahabad	Almora	Almora
		Ahmedabad	Ahmedabad	Ahmedabad	Ahmedabad	Bhuvaneswar	Bhuvaneswar
		Baroda	Baroda	Ahmednagar	Ahmednagar	Darjiling (North Bengal)	Darjiling (North Bengal)
			Nanded	Mumbai	Mumbai	Goa	Goa
			Kolkata	Baroda	Baroda	Hyderabad	Hyderabad
			Hyderabad	Nashik	Nashik	Indore	Indore
			Ernakulum	Jharkhand	Jharkhand	Jarkhand (Chattisgad)	Jarkhand (Chattisgad)
			Allahabad	Pune	Pune	Kerla	Kerla
			Goa	Almora	Almora	Kolkata	Kolkata
				Kerala	Kerala	Mumbai	Mumbai
				Indore	Indore	Nanded	Nanded
				Goa	Goa	Nashik	Nashik
				Varanasi	Varanasi	Pune	Pune
				Bhuvaneswar	Bhuvaneswar	Varanasi	Varanasi
					Darjiling	-	Delhi
							Banglore

Though the competition is yet to reach all states of the country, the overwhelming response from students and teachers is an encouraging indicator that in the near future it will reach all parts of the country.

We now turn our attention to three major aspects of the competition viz. Rules for the Competition, Curriculum for the competition and Sample problems of the competition.

3.4. **Rules for the Competition.** The following set of rules have been laid down for the competition :

(1) A three hour competition with maximum score 100
(2) Questions of three types: Objective (Multiple Choice), Short Answer Problems (Less Difficult) and Long Answer Problems (More Difficult)
(3) Meant for second year undergrad students, but interested students of lower standards may also appear
(4) First, Second and Third Prizes and several Cheer Prizes
(5) All participants would get certificates
(6) Prize winners to be invited for a Nurture Camp
(7) Spot entries are allowed

3.5. **Topics for the Competition.** As mentioned earlier, in India, there is no common curriculum for all undergrad students. Every University has the autonomy to frame their own curriculum. Therefore it is difficult to set up a common set of topics for the competition. However, we decided include the following topics, because they typically characterize undergraduate mathematics:

(1) Calculus of one variable: Continuity and differentiability of a function of one real variable, integration (as anti derivative), elementary differential equations
(2) Matrices: Rank and determinant of a matrithe x, system of linear equations
(3) Coordinate geometry of two and three dimensions
(4) Elementary number theory: divisibility, modular arithmetic, Fermat's little theorem
(5) Elementary Combinatorics: Permutations and combinations
(6) Algebra: Polynomials-relation between roots and coefficients, sets, functions etc.
(7) General logical puzzles

4. Sample Problems and Analysis of Results

4.1. **Sample Problems.** We now present the sample problems, a few of each types for understanding the level of the questions that have appeared in the competition. We place on record that at this early stage of the competition, we have used various sources(generally not available to undergrad students in India) like [?], [?], for setting up the questions in the competition. Thus the problems posed in the competition are not necessarily new ones and here we have tried to cite the references of the original sources wherever possible. But the absence of a reference does not mean the originality on the part of author.

Problem 4.1 : Objective Type

(1) The value of $\lim_{n\to\infty} \frac{1\cdot 1! + 2\cdot 2! + \cdots + n\cdot n!}{(n+1)!}$
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) does not exist. [2 Marks

(2) Let $A = \begin{pmatrix} 1 & 2 & \dots & n \\ n+1 & n+2 & \dots & 2n \\ \vdots & \ddots & & \vdots \\ (n-1)n+1 & (n-1)n+2 & \dots & n^2 \end{pmatrix}$. Select any entry and call it x_1. Delete row and column containing x_1 to get an $(n-1)\times(n-1)$ matrix. Then select any entry from the remaining entries and call it x_2. Delete row and column containing x_2 to get $(n-2)\times(n-2)$ matrix. Perform n such steps. Then $x_1 + x_2 + \cdots + x_n$ is
A) n B) $\frac{n(n+1)}{2}$ C) $\frac{n(n^2+1)}{2}$ D) None of these. 2 Marks

Problem 4.2 : Short Answer Problem

(1) Let H be a finite set of distinct positive integers none of which has a prime factor greater than 3. Show that the sum of the reciprocals of the elements of H is smaller than 3. Find two different such sets with sum of the reciprocals equal to 2.5. [6 Marks

(2) Let $f : [0,1] \to [0,1]$ be a function defined as follows :
$f(1) = 1$ and if $a = 0.a_1a_2a_3\cdots$ is the decimal representation of a (which does not end with a chain of 9's), then $f(a) = 0.0a_10a_20a_4\cdots$. Discuss the continuity of f at 0.392. [6 Marks

Problem 4.3 : Long Answer Problem

(1) Let $p(x)$ be a polynomial with positive integer coefficients. You can ask the question: What is $p(n)$ for any positive integer n? What is the minimum number of questions to be asked to determine $p(x)$ completely? Justify. [13 Marks

(2) Give an example of a function which is continuous at exactly two points and differentiable at exactly one of them. Justify your answer. [13]

(3) In an $m \times n$ matrix over N the only operations allowed are multiplying a row by 2 or subtract 1 from every member of a column. Can you reach a zero matrix in finitely many steps? Justify your answer. [12]

Readers are encouraged to tackle these problems and not to refer to the section 3.3 where we discuss the solutions of the problems. We shall not discuss the solutions of the objective type problems here. For some of the problems, we shall also get a chance to see a solution given by a students which is different from the official solution.
We shall now take a look at the overall performance of the students in the competition.

4.2. **Analysis of the Results.** The following table indicates the performance of students in last three years.

Students' Performance

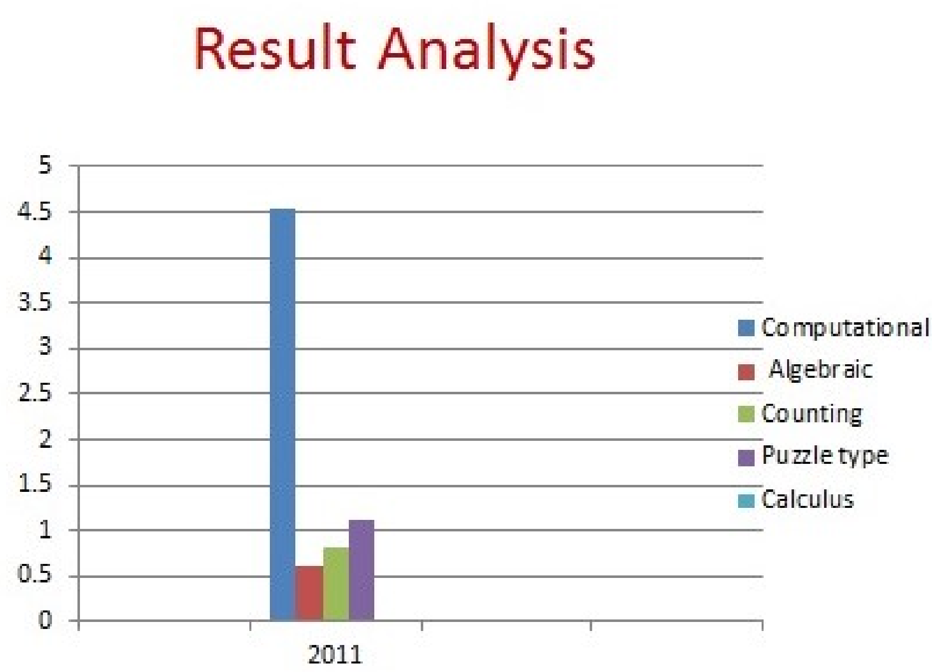
Year	Total No. of students	Marks above 30
December 2015	9041	37
January 2015	8327	61
January 2014	7672	59

The performance chart clearly reveals that the competition is very challenging for most of the students. There are several reasons for the low scores of students in the competition. A few main points causing the undesirable performances have been listed below:

(1) Students of undergraduate classes in India are generally not exposed to a competitive problem solving situation either in classrooms or in examinations

(2) The emphasis of the teaching-Learning processes at the undergrad level is unfortunately not inclined towards problem solving
(3) Not more than 5% of the students participating in Madhava competition have gone through Olympiad competitions. As mentioned earlier, most of the Olympiad students enter into professional courses and thus move outside the net of this competition.
(4) The University examinations are memory based, predictable and not oriented to test the problem solving capacities. For example, most of the universities ask to state and prove Lagrange's mean Value Theorem and many students end up solving it correctly. However, in Madhava competition we observed that not even 10% of the students are able to solve a problem based on mean value theorem.
(5) Students have fear for topics in Calculus, mainly due to $\epsilon - \delta$ definitions.

In the year 2011, we took a review of the performance of the students. It was revealed that even the better students have not done so well in Calculus problems. The following diagram clearly indicates this fact.



In fact, these students from higher academic bracket are typically past Olympiad students and they tend to capitalize on their Olympiad mathematics and solve problems on Number Theory and Combinatorics. With a view to encourage students to solve

Calculus problems, we then decided to diminish the level of Calculus problems. The performance on Calculus problems then got enhanced as can be seen from the table given below indicating the topic-wise distribution of top 25 students.

Year	Algebraic Problems	Puzzles	Calculus
December 2015	16	15	10
January 2015	18	0	08

The number of prizes offered every year varies according to the frequency of marks. The number of Cheer Prizes vary accordingly. The following table gives the information about the number of prizes awarded every year, the highest score in the competition and the cut-off for winning a prize.

Year	No. of Prize Winners	Highest Score	Cut-Off for a Prize
2010	16	69	25
2011	19	64	43
2012	25	74	40
2013	32	80	38
2014	10	74	47
January 2015	14	85	44
December 2015	8	92	47

The overall topper in all these years has scored 92 marks. We wish to mention that the student scoring 92 marks Mr. Pranav Nuti, has a strong Olympiad background and in fact has won a Bronze Medal in the IMO, 2013.

We now discuss the solutions of the problems stated earlier. We shall present official solution as well as elegant solutions given by students wherever possible.

4.3. **Solutions.**

(1) Let H be a finite set of distinct positive integers none of which has a prime factor greater than 3. Show that the sum of the reciprocals of the elements of H is smaller than 3. Find two different such sets with sum of the reciprocals equal to 2.5.

Solution: The given condition implies that every $n \in H$, n is of the form $n = 2^\alpha 3^\beta$, $\alpha, \beta \geq 0$. Since H is finite, $\exists k \in \mathbb{N}$ such that $\alpha \leq k$, $\beta \leq k$ for each $n \in H$. This implies [6 Marks

$$\sum_{n\in H} \frac{1}{n} \leq 1 + \sum_{i=1}^{k} \frac{1}{2^i} + \sum_{j=1}^{k} \frac{1}{3^j} + \sum_{i=1}^{k}\sum_{j=1}^{k} \frac{1}{2^i 3^j}$$

$$= 1 + \sum_{i=1}^{k} \frac{1}{2^i} + \sum_{j=1}^{k} \frac{1}{3^j} + \left(\sum_{i=1}^{k} \frac{1}{2^i}\right)\left(\sum_{j=1}^{k} \frac{1}{3^j}\right)$$

$$= \left(1 + \frac{1}{2} + \cdots + \frac{1}{2^k}\right)\left(1 + \frac{1}{3} + \cdots + \frac{1}{3^k}\right)$$
$$= (\frac{1 - \frac{1}{2^{k+1}}}{1 - 1/2})(\frac{1 - \frac{1}{3^{k+1}}}{1 - 1/3}) < \ (\frac{1}{1/2})(\frac{1}{2/3}) = 2(\frac{3}{2}) = 3.$$

Let $H = \{1, 2, 3, 4, 6, 8, 12, 24\}$. Then $\sum_{n \in H} \frac{1}{n} = 2.5.$

Let $H = \{1, 2, 3, 4, 6, 8, 12, 36, 72\}$. Then $\sum_{n \in H} \frac{1}{n} = 2.5.$

(2) Let $p(x)$ be a polynomial with positive integer coefficients. You can ask the question: What is $p(n)$ for any positive integer n? What is the minimum number of questions to be asked to determine $p(x)$ completely? Justify. [13]

Solution: Let $p(x)$ be a polynomial with positive integer coefficients say, $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k$. We can ask the question: what is $p(1)$? Let $p(1) = N$.
Then $N = a_0 + a_1 + a_2 + \cdots + a_k > \ a_i, \ \forall i$.
Then $p(N) = a_0 + a_1 N + a_2 N^2 + \cdots + a_k N^k$.
Now express $p(N)$ to base N, then i^{th} digit gives $a_i, \ \forall i$ which determines $p(x)$.

Alternatively, one may ask questions (i) what is value of $p(10)$? and (ii) what is value of $p(10^n)$, ? where n is the number of digits of $p(10)$. These two questions also determine the polynomial completely. The reader can check that the coefficients of $p(x)$ can be determined from the answers to these questions. Also, it is easy to prove that one question is not enough.

(3) Give an example of a function which is continuous at exactly two points and differentiable at exactly one of them. Justify your answer. [13] **Solution:** Define the function $f : \mathbb{R} \to \mathbb{R}$ thus:

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ x^3 & \text{if } x \text{ is irrational} \end{cases}$$

We show that f is continuous only at 0 and 1, and differentiable only at 0. For this, consider a real number a. Then as $x \to a$ through rational values, $f(x) = x^2 \to a^2$, and as $x \to a$ through irrational values, $f(x) = x^3 \to a^3$. So the limit $\lim_{x \to a} f(x)$ will exist if and only if the above two limits are equal i.e. if and only if $a^2 = a^3$ i.e. $a^2(a - 1) = 0$ i.e. $a = 0$ or $a = 1$. Thus f is continuous at 0 since $\lim f(x) = \lim x^2 = 0 = f(0)$. Similarly, f is continuous at 1. But when $a \neq 0, 1$, $\lim_{x \to a} f(x)$ does not exist; so f is discontinuous at a.

Next, let $g(x) = [f(x) - f(a)]/(x - a)$. Let a be rational. As $x \to a$ through irrational values, $\lim g(x) = \lim\{[x^3 - a^2]/(x - a)\}$ is not finite if $\lim[x^3 - a^2] \neq 0$ i.e. if $a^3 \neq a^2$ i.e. if $a \notin \{0, 1\}$. Hence $f'(a)$ does not exist (finitely) if $a \notin \{0, 1\}$. Let $a = 0$. Then $\lim_{x\to 0} g(x) = \lim_{x\to 0}[f(x)/x] = 0$. So $f'(0)$ exists and is 0.

But as $x \to 1$ through rational values, $\lim g(x) = \dfrac{x^2 - 1}{x - 1} = 2$, while as $x \to 1$ through irrational values, $\lim g(x) = \lim \dfrac{x^3 - 1}{x - 1} = 3$. Hence $f'(1)$ does not exist.

Let a be irrational. As $x \to a$ through rational values,

$\lim g(x) = \lim\{[x^2 - a^3]/(x - a)\}$

is not finite if $\lim[x^2 - a^3] \neq 0$ i.e. if $a^2 \neq a^3$ i.e. if $a \notin \{0, 1\}$. Hence $f'(a)$ does not exist.

There are several equivalent ways of defining such a function. For example the following function would also serve the purpose:

$$f(x) = \begin{cases} x^2(x-1) & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

(4) In an $m \times n$ matrix over N the only operations allowed are multiplying a row by 2 or subtract 1 from every member of a column. Can you reach a zero matrix in finitely many steps? Justify your answer. [12]

Solution: Yes, one method is as follows: Let $A = [a_{ij}]$ be the matrix. Let m be the minimum element in the first column C_1. In fact, let m occur s times i.e. let $m = a_{i_1 1} = \cdots = a_{i_s 1}$. We may *assume* that $m = 1$. For if $m \geq 2$, subtract 1 from each element of C_1 $m - 1$ times so that the minimum element in C_1 is 1.

Multiply each of the s rows $i_1, i_2, \ldots, i_s$ of A by 2. This forces the minimum element in C_1 to be 2. Subtract 1 from each element of C_1. The effect of these steps on C_1 is this: the s elements $a_{i_1 1}, a_{i_2 1}, \ldots, a_{i_s 1}$ of C_1 are still equal to 1, but the remaining elements of C_1 have all become *smaller* though they are all still ≥ 1. Hence in a finite number of steps *all* elements of C_1 will become 1. Then subtracting 1 from each element of C_1 makes C_1 a column of zeros.

Next make the second column C_2 a column of zeros as in the above. Note that the operations on C_2 have no effect on C_1 and C_1 *remains a column of zeros.* Hence in a finite number of steps A becomes the zero matrix.

5. Nurture Camp

The organisation of a nurture camp for prize winners of the competition is a distinctive feature of the competition. The duration of nurture camp varies from one week to ten days. The purpose of having a nurture camp is to provide insights to talented undergrads to advanced mathematics in the presence of distinguished mathematicians of the country. The camp also provides a unique opportunity to students to interact with their peers. The emphasis of the camp is on discussions and problem solving. The lectures in the camp are highly interactive and students are prompted to ask relevant questions. Some of the topics discussed in the camp are

(1) Killing-Hopf Theorem on the characterization of locally Euclidean surfaces.
(2) Poncelet's Theorem in Geometry
(3) Rational Points on Quadratic Forms
(4) Curvature of Curves and Surfaces
(5) Parametrization of Conics and Quadrics

Also information about pursuing a serious career in mathematics is given to students and they are told about various options in India and abroad for graduation in mathematics. Eminent mathematicians, Professor C. S. Rajan and Professor Raja Shridharan have been regularly participating in the nurture camp as resource persons. In the next section a feedback from Professor Rajan, especially about nurture camps has been recorded.

FIGURE 1. Nurture Camp 2016

6. Feedback

Any new initiative, especially a competition, needs to have a continuous feedback mechanism. The feedback helps in improving the academic as well as organizational matters associated with the event. Here, we record a sample feedback of Madhava Competition from both students and teachers involved in the competition.

We shall also include comments of two eminent mathematicians, Professor S. G. Dani and Professor C. S. Rajan who have been closely monitoring all the activities of Madhava Competition.

6.1. **Feedback of Students.** Here we record the feedback of two talented students who participated in the competition and won a prize.

Aditya Garg from Mumbai says:

The exam is a really good platform for talented, motivated and passionate students to showcase their talent in Mathematics. The problems were generally on the tougher side but really very interesting and well-designed and I thoroughly enjoyed solving them. I think this is one of the top-notch Mathematics competitions in the country and I really feel there should be more such competitions to motivate and find out talented students in Mathematics. And the nurture camp was even better. I got to interact with so many eminent professors, a number theorist and so many brilliant students. We had really good Mathematics sessions of problem solving and also gave presentations. We traded our knowledge of Mathematics with each other. I love Mathematics very much. And the camp was really very good. I still remember all the interactions and discussions we used to have with the teachers and the students in the camp.

Adway Gupta from Mumbai quotes:

As I spoke during the day of the felicitation, I believe that Madhava Mathematics Competition is a genuinely good medium for both the professional and amateur mathematician. I, as a student graduating with Physics, still found Madhava to be a very engaging paper. To be honest, the difficulty of MMC is definitely higher than your usual mathematics examinations. however, I believe that it is by design and that is exactly what makes it an extremely fun exam to sit for. The questions are designed in a way that knowing hardcore mathematical formulations isn't a prerequisite for doing well, a general mathematical intuition is enough. I think for the same reason I enjoyed taking the exam probably more than I did doing well in it.

6.2. **Feedback from Teachers.** A competition of this magnitude cannot be run without the support and help from colleagues across the country. In all the twenty one regions where the competition is being conducted, a dedicated teacher shoulders the responsibility as a Regional Coordinator. The comments of two regional coordinators have been included here.

My student and colleague Geetanjali Phatak has been closely associated with the competition right from its inception. Her reaction about the competition has been quoted below :

My relation with Madhava Mathematics Competition (MMC) has been multidimensional. I have contributed to MMC as a member of paper setting committee, coordinator of Pune Region and as a Tutor at Madhava nurture camp. As a mathematics teacher, I sincerely feel that students deserve an opportunity to face challenging problems in mathematics and MMC certainly fulfills this need. The group of students appearing for MMC is heterogeneous. However, majority of the students appearing for MMC are studying in colleges and as such do not have much of exposure. These students find the level of difficulty of problems posed at MMC as very high when compared to their university examinations. With the emergence of MMC, our undergraduate students have now started referring additional books other than the prescribed text books. We have also observed progress in terms of mutual discussions between the students as well as with the teachers. MMC helps students to generate interest in Mathematics. From the funds generated through registration fees, we can arrange various activities like guest lectures, summer workshops, etc. Nurture camp provides students an opportunity to discuss Mathematics with students from various parts of the country and experts from different institutes. I am sure that MMC will increase interest of students in Mathematics and I wish very best for the success of MMC.

Needless to mention that Madhava competition takes place in Kerala, the land where Madhava himself lived. The regional coordinator of Kerala region Dr. Aparna offered her comments:

Though Madhava Mathematics Competition began in 2010, center in Kerala region was established in the year 2012. In 2012, 869 students wrote MMC in 10 different centers in Kerala. Through years MMC became popular not only among Mathematics students in Kerala, but students from our neighboring state Tamilnadu is also writing exam at various sub-centers in Kerala. Last year the number of students who applied for the examination at various sub-centers in Kerala reached nearly 2000 and due to the increasing demand from the Mathematics community, we have increased the number of sub-centers to 14. For all registered students in Kerala region, a free training programme to enrich their Mathematical abilities is organized from last year onwards. Also we are giving motivational prizes to the state level winners.

We shall now present the views of two reputed mathematicians of the country who took keen interest in Madhava Competition and helped in enhancing the quality of the competition. It is important to record their opinion about this new initiative.

6.3. **Feedback from Mathematicians.** Professor S. G. Dani, a renowned mathematician, retired from TIFR, Mumbai and currently working at IIT(Powai) is well known for his contributions in Ergodic Theory. He is also a scholarly figure in the field of History of Mathematics. Professor Dani was the Chairman of NBHM when the proposal for the Madhava Competition was sent to NBHM for the financial assistance. He took keen interest in the proposal and recognized the potential in it. In all these years, he has been a constant source of encouragement and support. The author immensely values the comments offered by him. Professor Dani's thoughts are given below:

Competitions on a wider scale than in a limited learning group like in a school or college facilitate in generating interest in the subject and invigorate the studies as a whole. This applies especially to mathematics and has been fruitfully applied in various contexts and in various ways around the world. In India mathematical competitions at school level have flourished in the form of Olympiads, both in the main channel connected with the International Mathematical Olympiad and also outside it. A need was felt to introduce a similar activity at the undergraduate level, in colleges, in order to vitalize the interest in mathematics, especially among students with mathematical aptitude. A major step in this direction occurred when the S.P. College of Pune presented a proposal along the, spearheaded by Prof. Sholapurkar, to the National Board for Higher Mathematics for financial support. The Board recognized the immense potential in the proposal and after some tweaking of the original scheme, and gradual expansion the Madhava Mathematical Competition took shape. There has been a rapid expansion of the activity, both in numbers and the geographical spread around the country, within the short span of its existence. One would hope that it would become a defining feature of undergraduate studies in mathematics, and also expand to other countries.

Professor C. S. Rajan is a well known mathematician from TIFR, Mumbai. He works in Algebraic Number Theory. Prof. Rajan showed interest in the Competition right from the beginning. Especially, he liked the idea of having nurture camps for students. He participated in the nurture camps and gave deep insights to students. The points emphasized by Prof. Rajan have been recorded here.

(1) It helps in recognizing mathematical talent from the vast pool of undergraduate students in India.
(2) The nurture camp lets the selected students come into contact with their peers from across colleges and universities in India. Such contacts enable them to learn mathematics better; to encourage them and to maintain their interest in mathematics. Also it has the possibility of setting up long term (mathematical) friendships, which can be useful in future for collaborative work.
(3) The Indian undergraduate teaching is done mainly in colleges, many of which lack resources in terms of personnel, access to books and modern material.
(4) The nurture camp gives an opportunity for the students to come in contact with mathematics faculty drawn in from some of the the best places in the country.
(5) The structure of the nurture program has been designed so that it can illustrate the use of the undergraduate mathematics they are learning in their curricula to interesting mathematical problems accessible with the material they have learnt.
(6) Thus the camps give exposure for the students to some advanced mathematics and material that are not readily accessible; to expose them to the way mathematics is done by working mathematicians.
(7) It also helps in molding the tastes and interests of the students, naturally dictated by the faculty involved in the nurture camps. This is an extremely important but subtle aspect of the nurturing process.

(8) For the faculty, it allows them to come into contact with promising undergraduate students from across the country. This is quite satisfying for a faculty to be able to reach across to a wider cross section of students than he/she normally faces in their respective places.

7. Epilogue and Future Plans

The Madhava competition is a relatively new initiative in the area of Mathematics Competitions. Looking at the size of the country and heterogeneity in terms of language, academic background, geographical diversity, etc. it will take another few years to reach out to all parts of the country. Opening a center in the country mainly involves the appointment of a devoted mathematics teacher who would be willing to coordinate the event in the respective region. So far, the teachers have showed tremendous interest and enthusiasm in the activity and have extended their wholehearted support to the activity.

The competition has already reached major cities in India like Delhi, Mumbai, Kolkata, Ahmedabad, Hyderbad, Pune, Cochin, etc. This year the competition has reached to North-Eastern part of the country. Next year, we shall add a few more centers such as Tamilnadu, located in the southern part.
The setting of question paper for the competition is the most crucial aspect of the competition. We have tried our best to maintain a very high standard in setting up of questions. Though, so far we are borrowing questions from sources (and try to modify them wherever possible) that are not available to students, we would like to improve the situation by designing totally new questions.

The result analysis reveals that we need to improve the problem solving abilities of the students to a great extent. We plan to bring out a consolidated report on the competition and bring it to the notice of the government and university authorities. We hope that the curriculli of the universities would be designed so that the conceptual understanding is strengthened and as a result, students would be in position to tackle tough mathematical problems.

The nurture camp has been very useful for the students. We plan to extend the duration of the camp for better results. We shall also invite more students in the camp in coming years and propose to conduct more such camps in different parts of the country.

All in all, we have received a very positive feedback from students, teachers, mathematicians and math lovers in the country. We are sure that the activity will further flourish in the days to come and in turn, benefit the students community in the country. Organising such a competition has been a very rewarding experience and we sincerely hope that the activity would help in enhancing the mathematical aptitude of students of undergraduate level. On a larger sphere, the competition would certainly contribute, in its modest way, in generating a mathematically and logically strong human resource.

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